Consider a stream of payments (e.g. rents, bond coupons, etc)

$$C_1, C_2, C_3, \ldots, C_N$$

Let N = # of years

i = interest rate, nominal, annual

π = inflation rate of rents

PDV = present discounted value of payment stream

Let $C_1 = C_2 = C_3 = \cdots = C_N = C$

$$PDV = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^N}$$

$$= \frac{C}{i}\left\{1 - \left(\frac{1}{1+i}\right)^N\right\}$$ standard textbook formula

Case 1: constant stadium rental stream = \$10M/year with no rental inflation

If $\left.\begin{array}{l} i = .04/\text{yr} \\ N = 30 \text{ years}\end{array}\right\} \Rightarrow PDV = \left(\frac{10}{.04}\right)\left\{1 - \left(\frac{1}{1.04}\right)^{30}\right\}$

$$= (250)\{.6917\} = \boxed{\$172.92\text{M}}$$

This equals published CSAG estimate.

Case 2: Rental is \$10 M in year 1 and rises at rate $\pi = .03$ every year thereafter through year 30.

$$PDV = \frac{10}{1+i} + \frac{10(1.03)}{(1+i)^2} + \frac{10(1.03)^2}{(1+i)^3} + \cdots + \frac{10(1.03)^{29}}{(1+i)^{30}}$$

$$= \left(\frac{10}{1+i}\right)\left\{1 + \frac{1.03}{1+i} + \frac{(1.03)^2}{(1+i)^2} + \cdots + \frac{(1.03)^{29}}{(1+i)^{29}}\right\}$$

Now use the approximation: $\left(\frac{1+\pi}{1+i}\right)^N \approx \left(\frac{1}{1+i-\pi}\right)^N$

Proof: take $\ln$ of both sides + apply approximation $\ln(1+x) \approx x$ for small x [using 1^{st} order Taylor series expansion about 0]

$$\Rightarrow PDV = \left(\frac{10}{1+i}\right)\left\{1 + \underbrace{\frac{1}{1+i-\pi} + \frac{1}{(1+i-\pi)^2} + \cdots + \frac{1}{(1+i-\pi)^{N-1}}}_{= \left(\frac{1}{i-\pi}\right)\left\{1 - \left(\frac{1}{1+i-\pi}\right)^{N-1}\right\}}\right\}$$

$N = 30$
$\pi = .03$

Thus $PDV = \frac{10}{1+i}\left\{1 + \left(\frac{1}{i-\pi}\right)\left[1 - \left(\frac{1}{1+i-\pi}\right)^{N-1}\right]\right\}$

$N = 30$; $\pi = .03$; $i = .04 \Rightarrow i - \pi = .01$

$$\Rightarrow PDV = \left(\frac{10}{1.04}\right)\left\{1 + \left(\frac{1}{.01}\right)\left[1 - \left(\frac{1}{1.01}\right)^{29}\right]\right\} = (9.615)\left\{1 + \left(\frac{1}{.01}\right)[.2506]\right\}$$

$$= (9.615)\{26.066\} = \boxed{\$250.62\ M}$$